Enrollment No: ____________________ **Exam Seat No:**____________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Number Theory

Subject Code: 5SC04NUT1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 24/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

- **a.** Define: Prime numbers. **1**
- **b.** Define: Greatest integer function. **1**
- **c.** If P_n is the n^{th} prime number then $P_n \geq 2^{2^{n-1}}$. True or False. **1**
- **d.** Find highest power of 3 that divides 81!. **2**
- **e.** Calculate $\phi(360)$. **2**

Q-2 **Attempt all questions** **(14)**

- **a.** State and prove fundamental theorem of divisibility. **6**
- **b.** Let a and b are two integers, not both zero. Then a and b are relatively prime if and only if there exists integers x and y such that $1 = ax + by$. **4**
- **c.** State and prove relation between l.c.m and g.c.d. of two numbers. **4**

OR

Q-2 **Attempt all questions** **(14)**

- **a.** State and prove Euclidean algorithm. **6**
- **b.** If $k > 0$, then $\gcd(ka, kb) = k \gcd(a, b)$. **4**
- **c.** Prove that there are infinitely many prime numbers. **4**

Q-3 **Attempt all questions** **(14)**

- **a.** State and prove Unique factorization theorem for positive integer. **6**
- **b.** For any integer $n > 1$, prove that $\phi(n) = n - 1$ if and only if n is prime. **4**
- **c.** Find number of multiple of 7 among the integers 200 to 500. **4**

OR

Q-3 **Attempt all questions** **(14)**

- **a.** In usual notation prove that the function τ and σ are both multiplicative function. **6**
- **b.** Find remainder when sum $1! + 2! + 3! + \cdots .. + 99! + 100!$ Is divided by 12. **4**
- **c.** Check whether the integer 1571724 is divisible by 9 and 11. **4**

SECTION – II

Q-4 **Attempt the Following** **(07)**

- **a.** Define: Mobius function. **1**
- **b.** State Euler's theorem. **1**
- **c.** Define: Primitive root. **1**
- **d.** Define: Pythagorean triplet. **1**
- **e.** Define: Pell's equation. **1**
- **f.** How many solutions of $x^d - 1 \equiv 0\,(mod p)$? Where $p$is prime and $d|(p-1)$. **1**
- **g.** For any positive integer a and p, $a^p \equiv a(mod p)$. True or false. **1**

Q-5 **Attempt all questions** **(14)**

- **a.** State and prove Chinese remainder theorem. **7**
- **b.** Solve: $18x \equiv 30\ (mod\ 42)$. **4**
- **c.** Find two primitive roots of 10. **3**

OR

Q-5 **Attempt all questions** **(14)**

- **a.** State and prove Wilson's theorem **7**
- **b.** Solve the following system of congruence. **4**
  $2x \equiv 1\ (mod\ 5)$ and $3x \equiv 1\ (mod\ 7)$.
- **c.** In usual notation prove that p_k and q_k are relatively prime for $1 \le k \le n$. **3**

Q-6 **Attempt all questions** **(14)**

- **a.** State and prove Lagrange's theorem. **6**
- **b.** Solve: $172x + 20y = 1000$. **4**
- **c.** Prove that the value of any infinite continued fractions is an irrational number. **4**

OR

Q-6 **Attempt all Questions**

- **a.** State and prove Fermat's last theorem. **9**
- **b.** Prove that any rational number can be written as a finite simple continued fractions. **5**